Stats 2 - June 2008

① a) 2 by 2, so need Yates Correction

Expected

	A	No A
H	44	66
L	36	54

Yates X_2 $\frac{(|O-E|-0.5)^2}{E}$

	A	No A
H	1.2784	0.8523
L	1.5625	1.0417

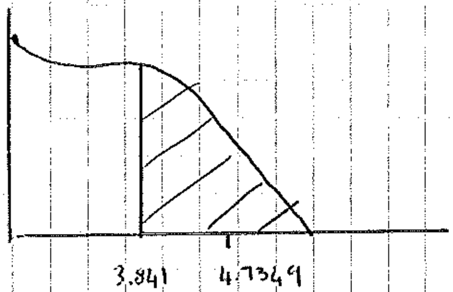
Total $X_2 = 4.7349$ (TEST STATISTIC)

H_0: No association between Asthma + Traffic (Independent)

H_1: Association between Asthma + Traffic (Associated)

$V = (2-1)\times(2-1) = 1$

CRITICAL VALUE $X_{2\,(1)}$ 5% : 3.841



$4.7349 > 3.841$

∴ Reject H_0

Evidence at 5% level to suggest an association between incidents & Asthma and the traffic volume.

b) 44 were expected, 52 were recorded

∴ More than expected had Asthma.

② a) $X \sim Po(6)$

$P(X=8) = \frac{e^{-6}(6^8)}{8!} = 0.10325..$

b) $Y \sim Po(3^2)$

i) $\lambda = 3^2 = 9$

ii) $P(Y>9) = 1 - P(Y \leq 9)$

$= 1 - 0.5874$ (from tables)

$= 0.4126$

c) i) $T \sim Po(15)$

ii) $P(T \leq 20) = 0.9170$ (from tables)

iii) $P(T \geq 21) = 1 - P(T \leq 20) = 1 - 0.9170 = 0.083$

For 4 out of 6, use Binomial

$\Rightarrow {}^6C_4 \times (0.083)^4 \times (0.917)^2 = 0.0005986...$

③ $H_0: \mu = 34.5$

$H_1: \mu \neq 34.5$ (2 tailed test)

$\sigma = 2.5$

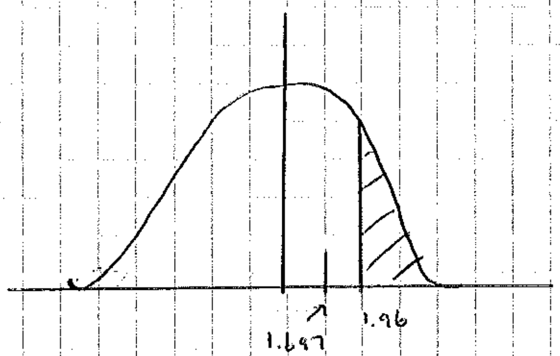
$\bar{x} = 35.1$

$n = 50$

we know σ and $n = 50$, so use Z distribution due to CLT

TEST STATISTIC: $\dfrac{35.1 - 34.5}{2.5/\sqrt{50}} = 1.697...$

CRITICAL VALUE: Z, 5%, 2 tailed $= \pm 1.96$

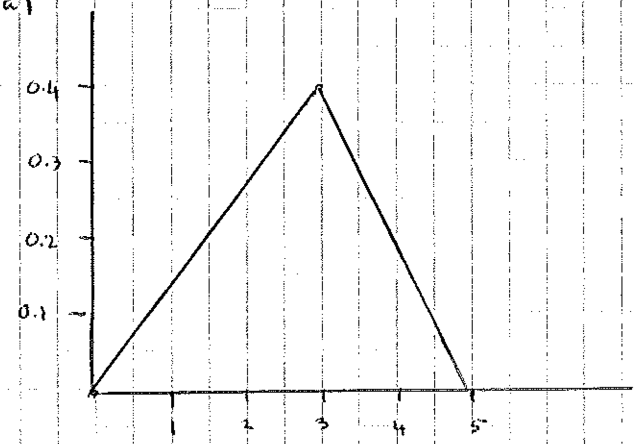


$1.697 < 1.96$

$\therefore$ Accept H_0

Not enough evidence at 5% level to suggest mean weight has changed.

④ a)



b) i) $P(T \leq 2)$

Use area of Δ = $\frac{1}{2} \times 2 \times \frac{4}{15}$

$= \frac{4}{15}$

ii) $P(2 < T < 4)$

$P(T < 2) = \frac{4}{15}$

$P(T > 4) = \frac{1}{2} \times 1 \times \frac{1}{5} = \frac{1}{10}$

$\therefore P(2 < T < 4) = 1 - \left(\frac{4}{15} + \frac{1}{10}\right)$

$= \frac{19}{30}$

c) $E(T) = \int_0^3 t f(t) + \int_3^5 t f(t)$

$= \int_0^3 2t^2/15 + \int_3^5 t - t^2/5$

$= \left[\dfrac{2t^3}{45} \right]_0^3 + \left[\dfrac{t^2}{2} - \dfrac{t^3}{15} \right]_3^5$

$= \left(\dfrac{54}{45} - 0 \right) + \left(\dfrac{25}{2} - \dfrac{125}{15} - \dfrac{9}{2} + \dfrac{27}{15} \right)$

$54/45 + 22/15 = 8/3 = 2\,2/3$

(5) a) i) $\Sigma x = 31.9$ $\quad \bar{x} = 31.9/10 = 3.19$

$\Sigma (x - \bar{x})^2 = 1.849$ $\quad s^2 = \dfrac{1.849}{9} = 0.20544\ldots$

$n = 10$

$v = 10 - 1 = 9$

t value (9) 99% (2 tailed) = 3.250

99% CI $= 3.19 \pm 3.250 \times \sqrt{\dfrac{0.20544}{10}}$

$= 3.19 \pm 0.46583$

$= (2.724, 3.656)$

ii) Claim seems reasonable as 3.5g is comfortably in the confidence interval

b) $0.01 \times 200 = 2$

(6) $H_0: \mu = 3.8$

$H_1: \mu > 3.8$ (1 tailed test)

Don't know σ, and $n = 7$ so use t

Data (from calc):

$\Sigma x = 28.7$ $\quad \bar{x} = 4.1$

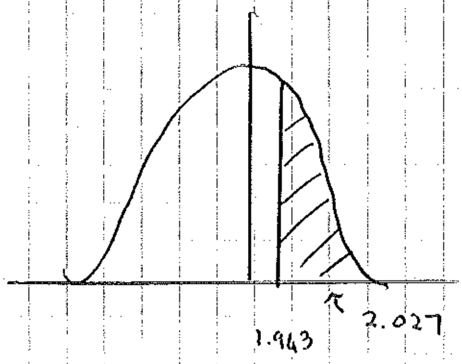
$\Sigma x^2 = 118.59$ $\quad s = 0.39157\ldots$

$n = 7$ $\quad s^2 = 0.15333\ldots$

$v = 7 - 1 = 6$

TEST STATISTIC : $t = \dfrac{4.1 - 3.8}{0.39157/\sqrt{7}} = 2.027...$

CRITICAL VALUE : $t_{(6)}$ 5%, 1 tailed $= 1.943$



Assumption: Cholesterol levels normally distributed

$2.027 > 1.943$

$\therefore$ Reject H_0

There is evidence at 5% level to support belief that level of cholesterol is higher than manager's claim

(7) a) i) Mean : $E(X) = 5 \times 0.1 + 15 \times 0.2 + 25 \times 0.3 + 35 \times 0.4$

$= 25$

$E(X^2) = 5^2 \times 0.1 + 15^2 \times 0.2 + 25^2 \times 0.3 + 35^2 \times 0.4$

$= 725$

Variance $= 725 - 25^2 = 100$

$\therefore$ sd $= \sqrt{100} = 10$

ii) $E(C) = 10 \times E(X) + 5 = 10 \times 25 + 5 = 255p$

b) $Var(X) = 75.25 - 8.35^2 = 5.5275$

$Var(T) = 0.4^2 \times 5.5275 = 0.8844$

(8) a) $P(X < 0) = F(0) = \dfrac{0+1}{k+1} = \dfrac{1}{k+1}$

b) For LQ (q_1), $F(q_1) = 0.25$

$\dfrac{q_1 + 1}{k+1} = 0.25$

$\rightarrow q_1 + 1 = 0.25(k+1)$

$\rightarrow q_1 = 0.25(k+1) - 1$

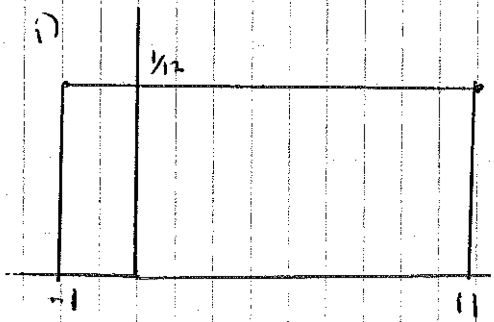
c) $f(x) = F(x)$ differentiated

If $y = \dfrac{x+1}{k+1}$, ~~dWan~~ $y = \dfrac{x}{k+1} + \dfrac{1}{k+1}$

$dy/dx = \dfrac{1}{k+1} = f(x)$ For other values

$\rightarrow f(x) = \begin{cases} \dfrac{1}{k+1} & -1 \le x \le k \\ 0 & \text{Otherwise} \end{cases}$

d) i)



ii) $E(X) = \dfrac{11 + -1}{2} = 5$

$Var(X) = \frac{1}{12}(11 - (-1))^2$

$= 12$

iii) $P(q_1 < X < E(X))$

$q_1 = -1 + \frac{1}{4}(11 - (-1)) = 2$

$E(X) = 5$

$P(2 < X < 5) = (5 - 2) \times \frac{1}{12} = 0.25$

From Diagram